Production and Geologic Time-scale Compaction in Rigid Grain-rich and Ductile Grain-rich Sands

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Acknowledgements: F.H.K. Rambow, T.R. Taylor, D.K. Love, M.M. Arasteh, N. Sun, and H. Reddy

Modelling Stress Dependent Compressibility

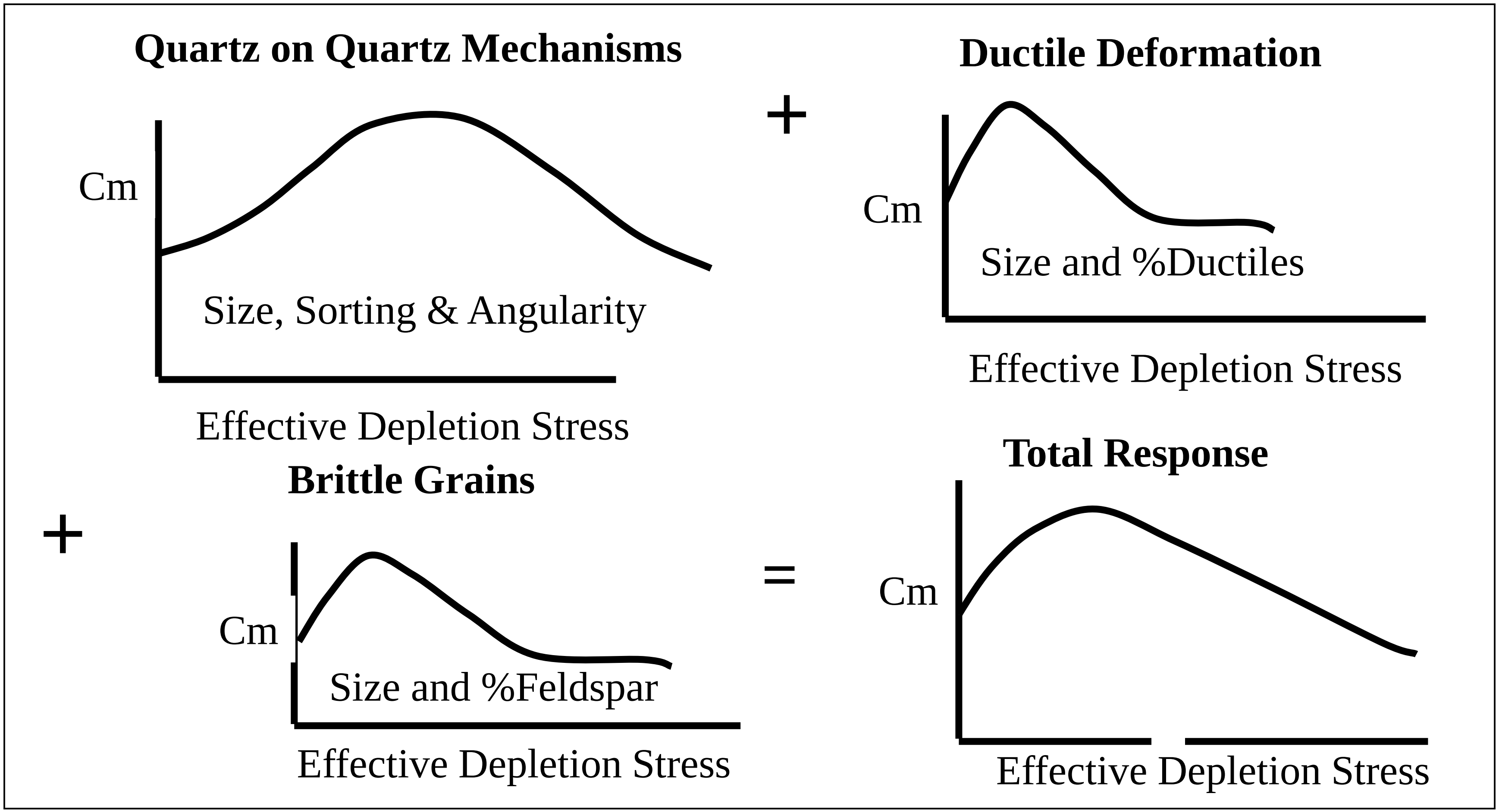
- Sand pack studies suggest that a model for stress dependent pore volume compressibilty must take into account:

- The number of compacting sites – increasing Cm as volume of less competent grains increases.

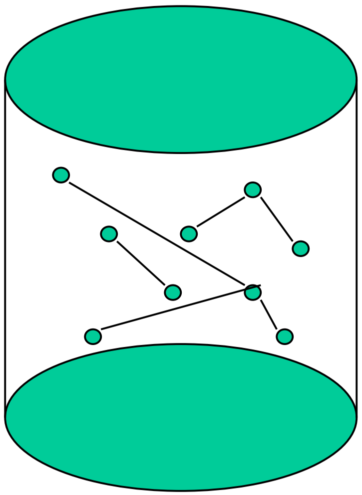
- The shift of peak compressibility to lower stress as the number of compacting sites increases.

- Observed asymptotic behavior at very high effective depletion stress.

Predicting Compressibility: Starting Assumptions



Simplest (Trivial) Statistical Model



Add Compaction Sites

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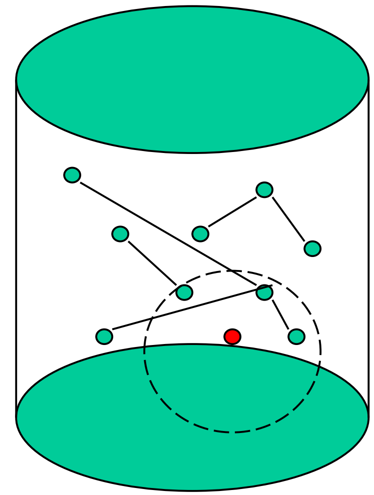
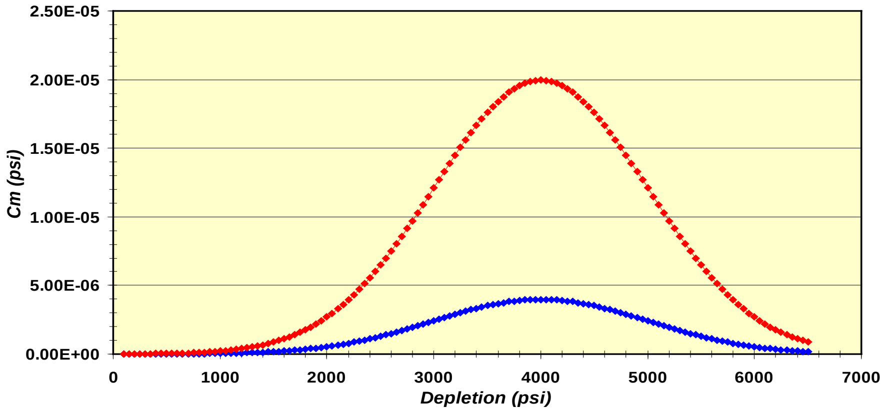
Add Links – Probability of Forming a Link is Proportional to the Fraction of Sites with that Number of Links



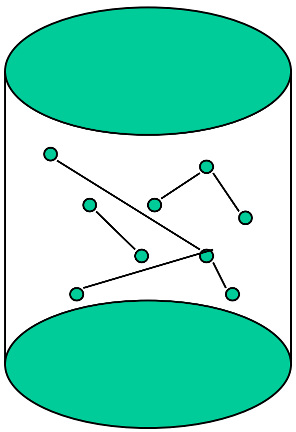
Leads to model with Linear Scaling



$$\frac{\Delta n(N,j)}{n(N,j)} = \frac{\Delta N}{N}$$

The initial known network consists of nodes with multiple connections to other nodes. The statistics of the network are described by n(N,j) the number of nodes with j connections when there are N total nodes in the network.

More Complex Statistical Model



Add Compaction Sites

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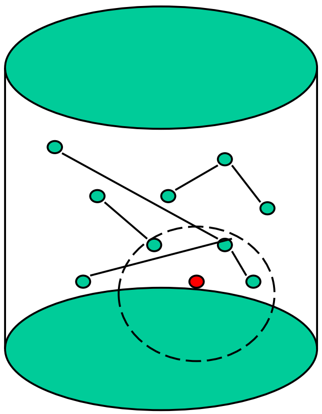
Add Links – Probability of Forming a Link is Proportional to the Fraction of Sites with that Number of Links
Scramble Existing Links – The new number of sites increases proportional to the fraction of sites with one less link and decreases proportional to the fraction with the same number of links



Basic statistical idea behind the network growth model. Adding a bond to an existing node promotes it to the next n(N,j) class increasing the value of that class by one and reducing the class it left by one.

Derivation

We now start with an expression for *n* which includes the possibility of adding bonds above a simple scaling (i.e., includes the k(n,j) terms) with scaling factors A, B, C to be determined).

$$n(N+\Delta N, j) = n(N, j) + A \cdot k\left(n, j-1\right) n\left(N, j-1\right) - B \cdot k\left(n, j\right) n\left(N, j\right) + C \cdot n(N, j) \qquad (1)$$

As discussed above for k(n,j) identically equal to zero the base case of self-similar scaling should be retrieved:

$$n(N+\Delta N, j) = n(N, j) + \frac{\Delta N}{N} n\left(N, j\right) \qquad (2)$$

Since the equality must be true for all $n(N, j)$, we obtain:

$$C = \frac{\Delta N}{N} \qquad (3)$$

Derivation

Remembering that with k(n,j) identically equal to one all the nodes have added an additional bond above the base case, i.e., all nodes with j-1 bonds will now have j bonds and similarly all nodes with j bonds now have j+1 bonds, etc. The base case should therefore still be obtained. However, it now depends on (j-1) instead of j . Therefore, for k identically equal to one, we obtain:

$$n(N+\Delta N, j) = n(N, j) + \frac{\Delta N}{N} n\left(N, j-1\right) \tag{4}$$

Substituting (4) into the equation (3) and collecting terms:

$$(A - \frac{\Delta N}{N}) \cdot n\left(N, j-1\right) - (B - \frac{\Delta N}{N}) \cdot n\left(N, j\right) = 0 \tag{5}$$

Which again must be satisfied for all n , and implies that both terms must be identically zero or that:

$$A = B = \frac{\Delta N}{N} \tag{6}$$

Derivation

Substituting for A and B in equation (5) gives:

$$n(N+\Delta N, j) = n(N, j) + \frac{\Delta N}{N} n(N, j) + \frac{\Delta N}{N} \cdot \left[k(n, j-1) n(N, j-1) - k(n, j) n(N, j)\right] \qquad (7)$$

Expanding the term in the brackets and rearranging terms gives:

$$N \cdot \frac{n(N+\Delta N, j) - n(N, j)}{\Delta N} = \left[k(n, j) n(N, j) - \frac{\partial\left(k(n, j) \cdot n(N, j)\right)}{\partial j} - k(n, j) \cdot n(N, j)\right] + n(N, j) \qquad (8)$$

Canceling terms and in the limit of small ΔN, results in:

$$N \cdot \frac{\partial n(N, j)}{\partial N} + k(n, j) \cdot \frac{\partial n(N, j)}{\partial j} = n(N, j)\left(1 - \frac{\partial k(n, j)}{\partial j}\right) \qquad (9)$$

Interpretation of the Equation

$$N \cdot \frac{\partial n(N,j)}{\partial N} + k(n,j) \cdot \frac{\partial n(N,j)}{\partial j} = n(N,j)\left(1 - \frac{\partial k(n,j)}{\partial j}\right) \qquad (9)$$

This is an inhomogeneous first-order transport equation which are usually interpreted as conservation laws. It describes motion only in one direction, the term *k(n,j)* determines the speed and direction of the transport. The inhomogeneous term on the right hand side accounts for the growth of the network. The speed will be related to the function k and its dependence on n and j as the network grows. To examine this dependence, the solutions of the equation up to first order expansions of k in terms on n and j are examined.

Solutions of the Equation –Linear Models

It is now assumed that the higher the number of bonds associated with a node the more likely it is to add a bond, i.e., that $k(n, j) = k_0 + k_1 \cdot j$. This gives:

$$n(N, j) = \eta^{1-k_1} D\left(\frac{j - \frac{k_0}{k_1}\left(\eta^{-k_1} - 1\right)}{\eta^{-k_1}}\right) \qquad (10)$$

The width of the distribution increases with increasing η. Nodes associated with larger values of j have a higher speed. Solving the equation with a j^2 term emphasizes this effect and allows the distribution to skew.

Linear Models – Stress Dependence

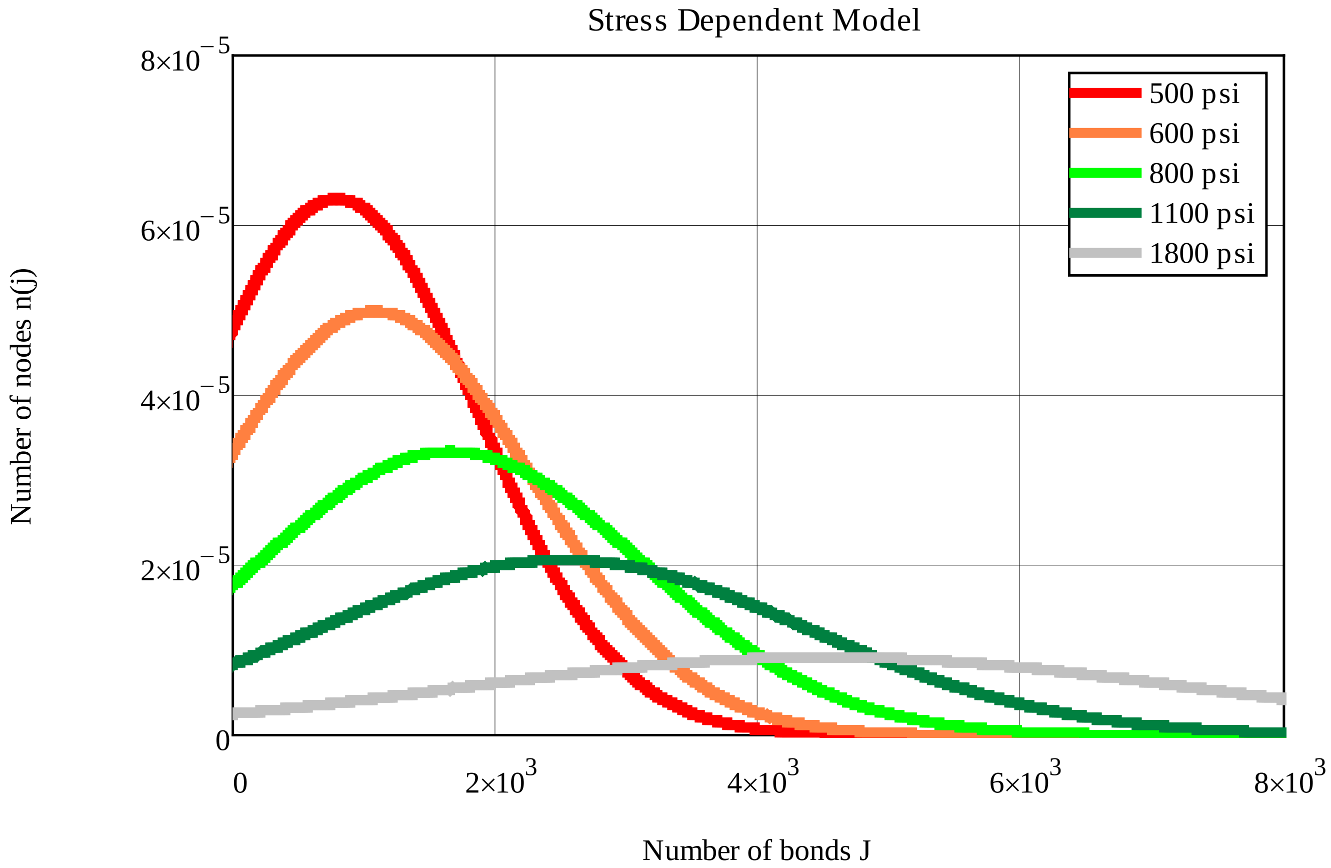
The linear solution of the transport equation establishes a relationship between the number of nodes n with j bonds and the fractional change in the total number of nodes η

$$n(N, j) = \eta^{1-k_1} D\left(\frac{j - \frac{k_0}{k_1}\left(\eta^{-k_1} - 1\right)}{\eta^{-k_1}}\right) \tag{10}$$

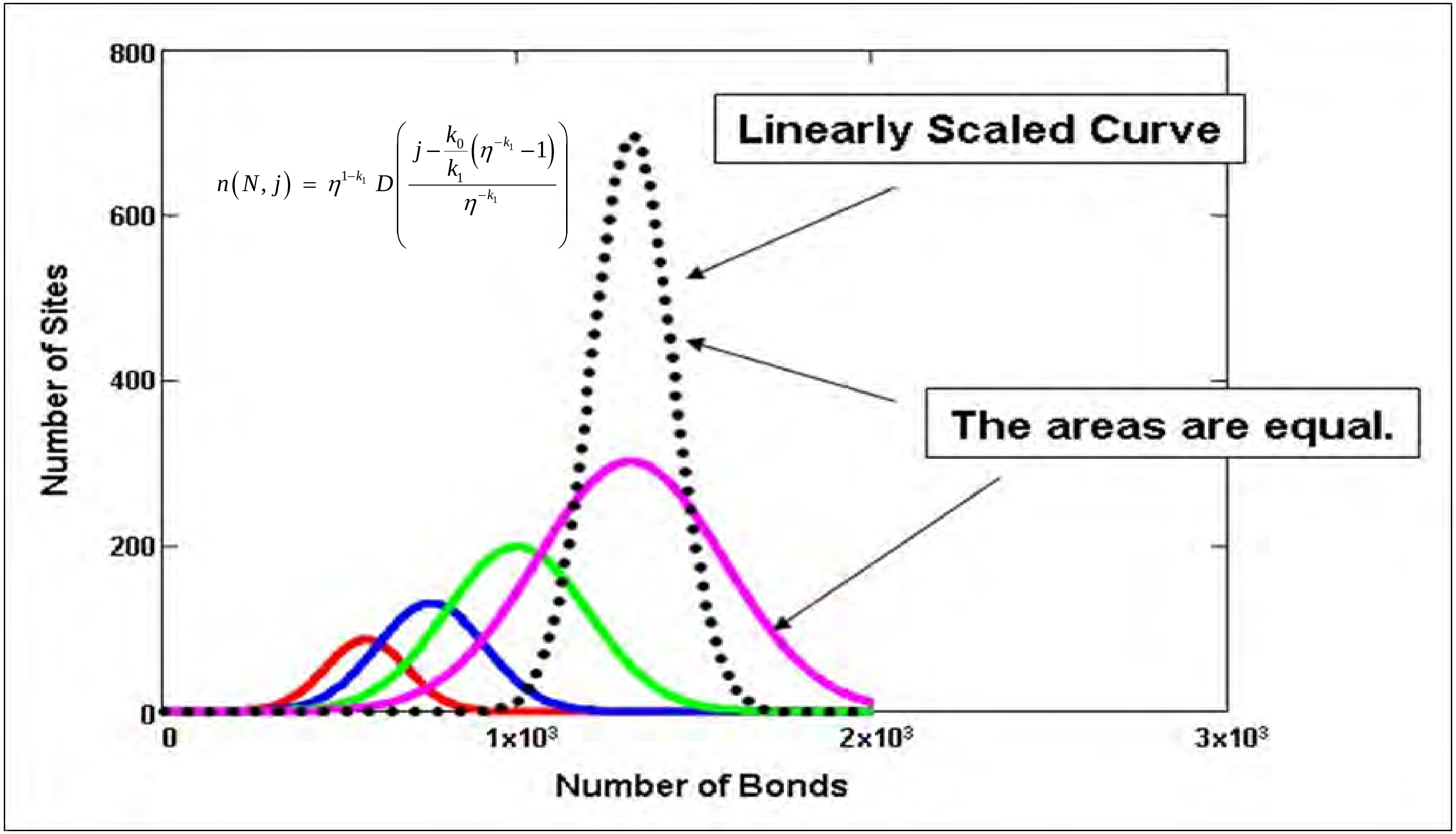
This relationship can be inverted to establish a relationship between a change in the number of bonds and the equivalent η.

$$\eta = \left(\frac{k_0 + k_1 \cdot \Delta \cdot j}{k_0}\right)^{\frac{1}{k_1}} \tag{11}$$

Linear Models – Stress Dependence

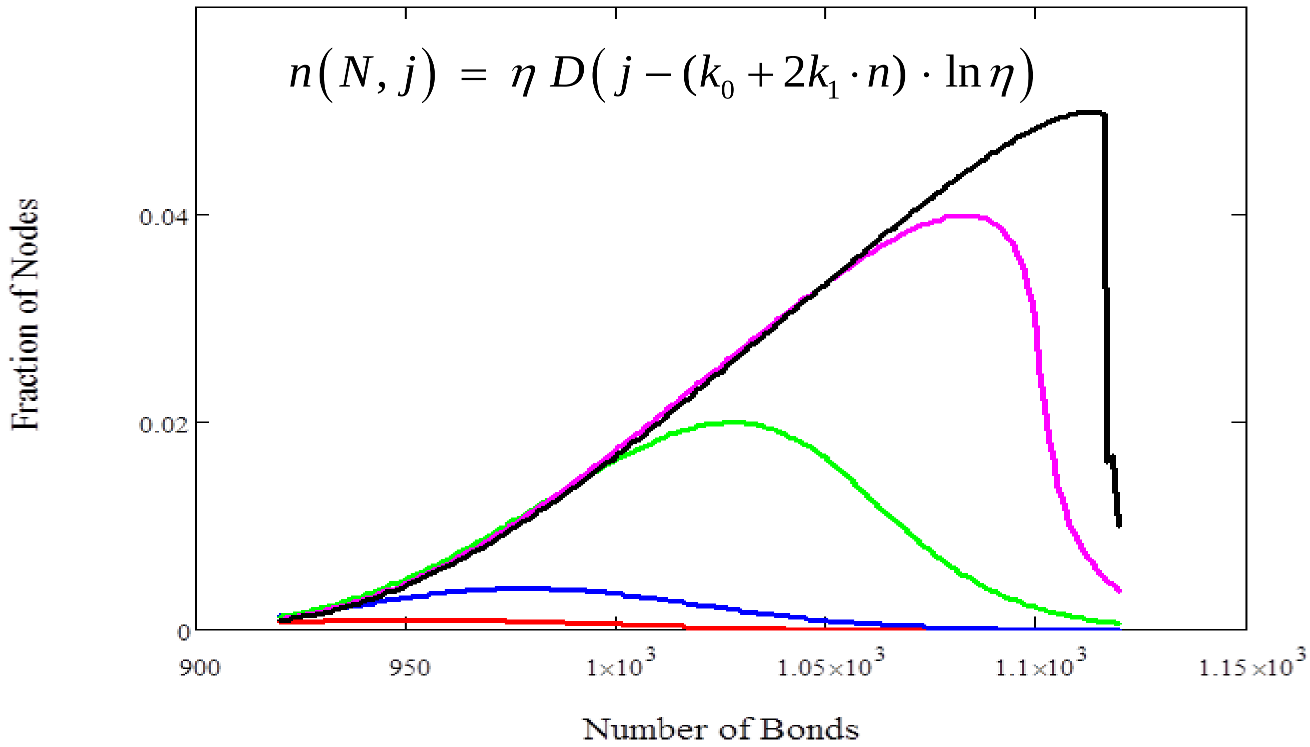


Linear Models



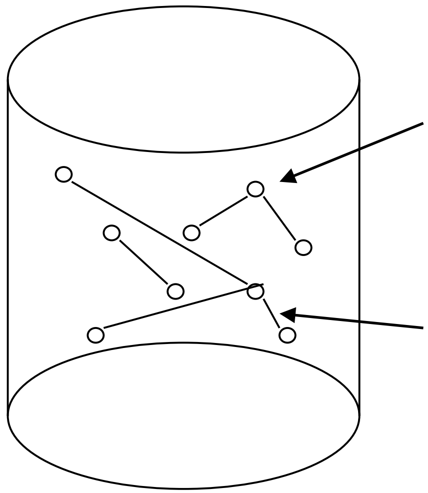
Solution for $k(n, j) = k_0 + k_1 j$ (solid lines) compared to the solution with constant k(n , j) = k0 (dotted line). The solution broadens because of the k_1 term and translates due to k_0 while it grows. The higher the value of j, the faster the velocity. Notice that the curve does not skew i.e. it's a linear model

Nonlinear Models



Solutions to the non-linear transport equation. Non-linear effects result in the sudden decrease in the fraction of nodes at a high connection number. This effect is similar to the breaking of water waves.

Modeling Compressibility - Definitions



Compaction Site is an Elemental Compactor
- Not influenced by its environment

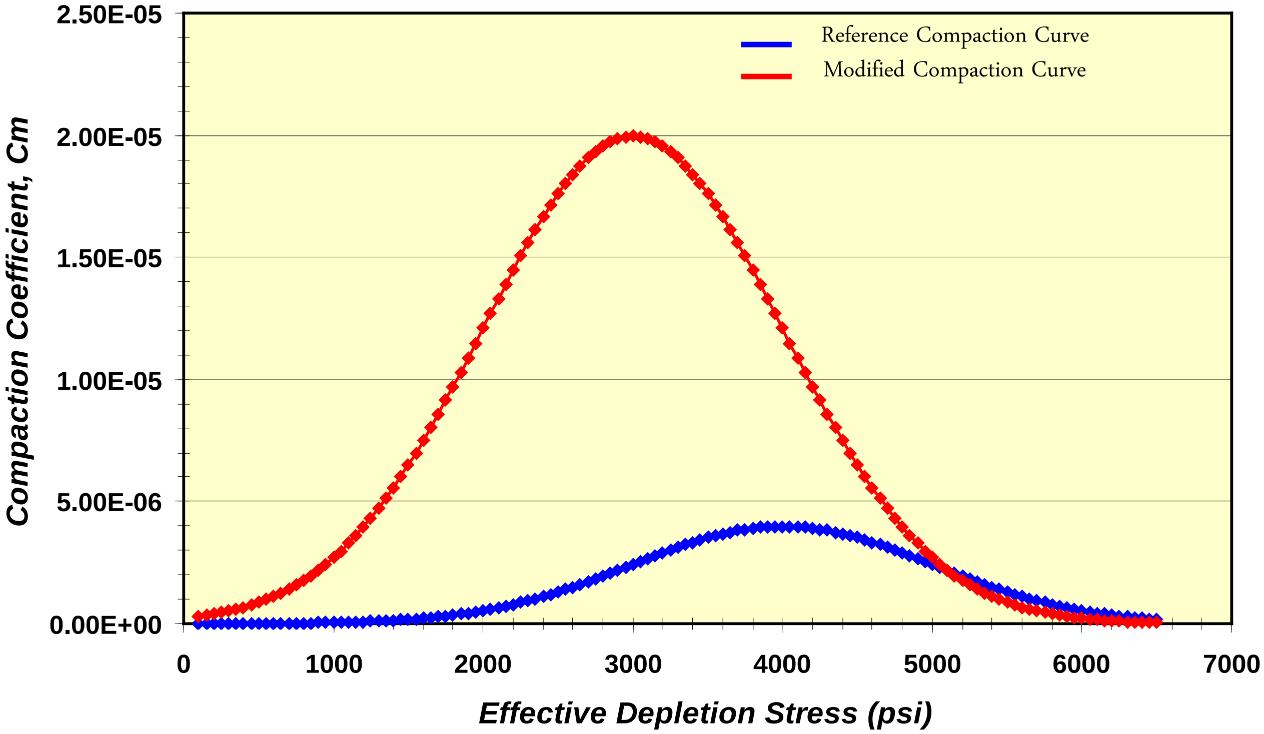
A Compaction Site is Linked to Another Site
- If it lowers the stress at which that site compacts

The Larger the Number of Links
The Lower the Critical Stress for Compaction

Modeling compaction involves mapping a network to parameters related to compaction. The nodes are the source of the displacements, i.e., grains and the bonds are mapped into force chains. The larger the number of force chains the higher the stress at which the grain displaces.

Predicting Compressibility : Starting Assumptions

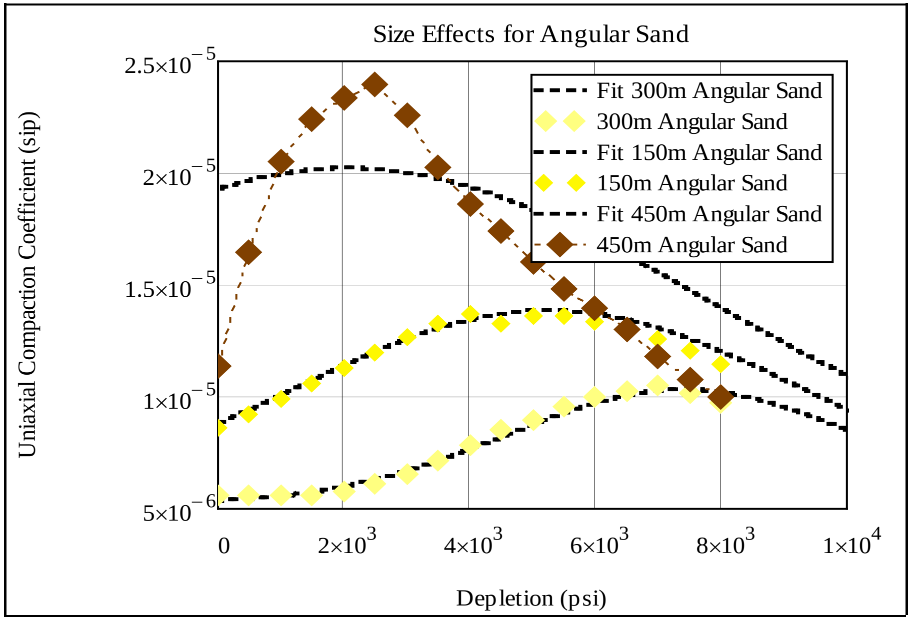
Adding compacting sites will cause the magnitude of the compressibility to ***increase*** and will ***decrease*** the stress required to achieve maximum Cm

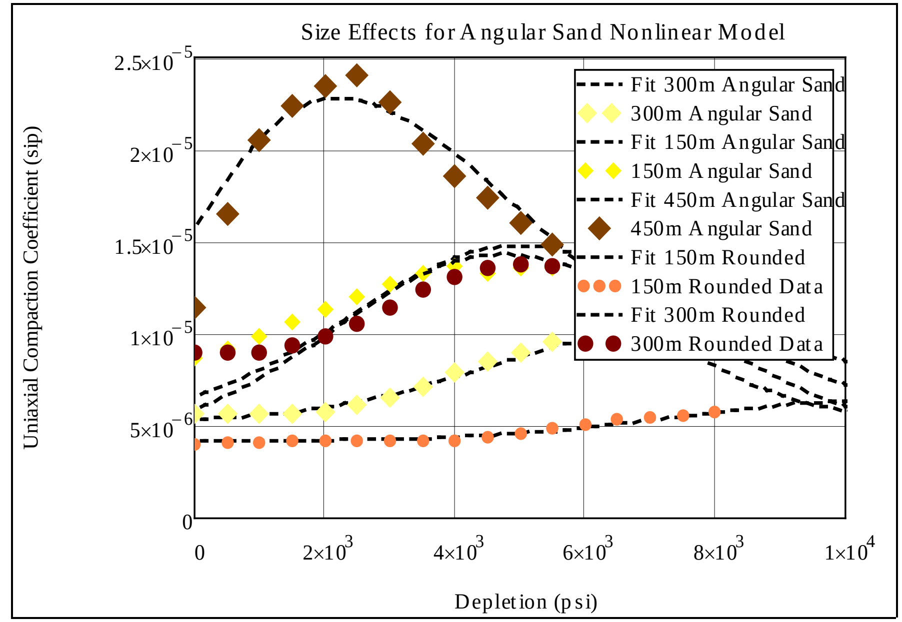


Sand Pack Experiments – Group I and Group II

To determine the form of k(n,j) for different compaction mechanisms, controlled experiments were performed in which a single textural or mineralogical feature was systematically varied. Uniaxial strain measurements were made at an initial stress of 1250 psi confining stress. Examples included the volume of ductile (clay or phylite) or brittle grains (feldspar), grain size, grain angularity, sorting, etc. The details of the thin section observations associated with these measurements is reported elsewhere. Fits were then performed to extract the relevant model parameters for two different packings (Group I and Group II).

Fits for Group 1 - Grain Size



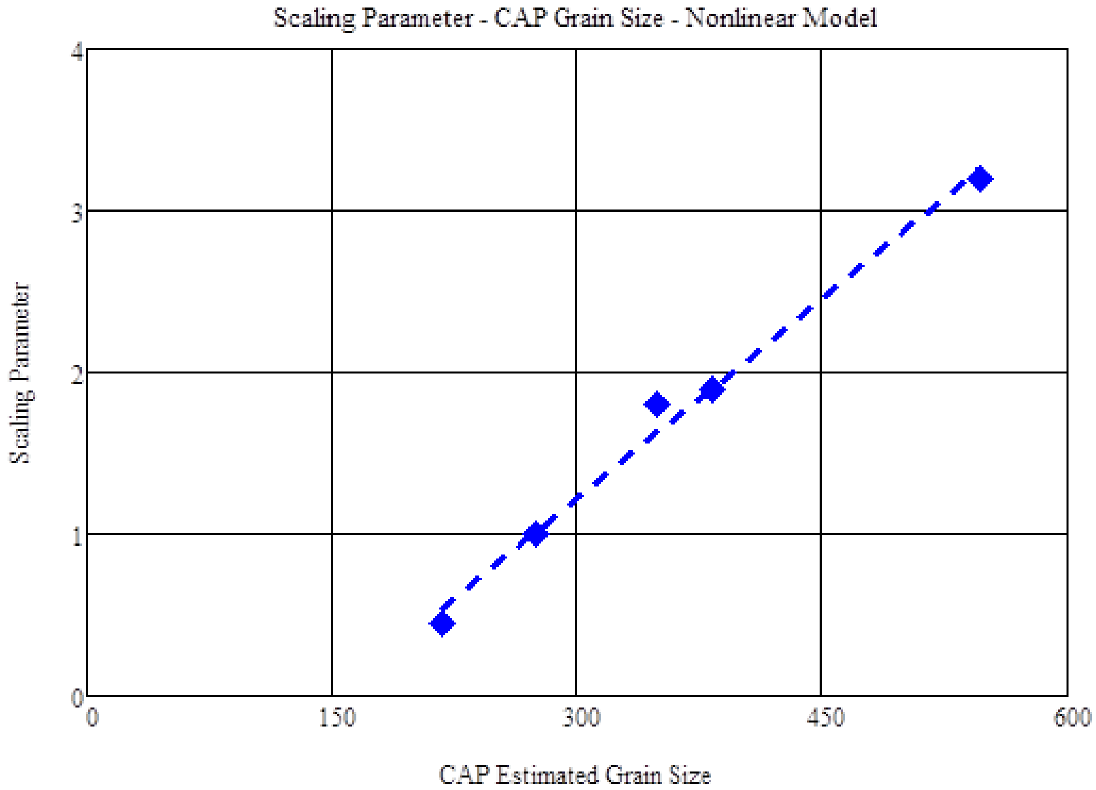


Size effects for the angular sand. The fit fails to capture the skewed nature of the compressibility at the largest grain size.

The non-linear fits to all of the grain size data. Notice the much improved fit to the 450 micron sample..

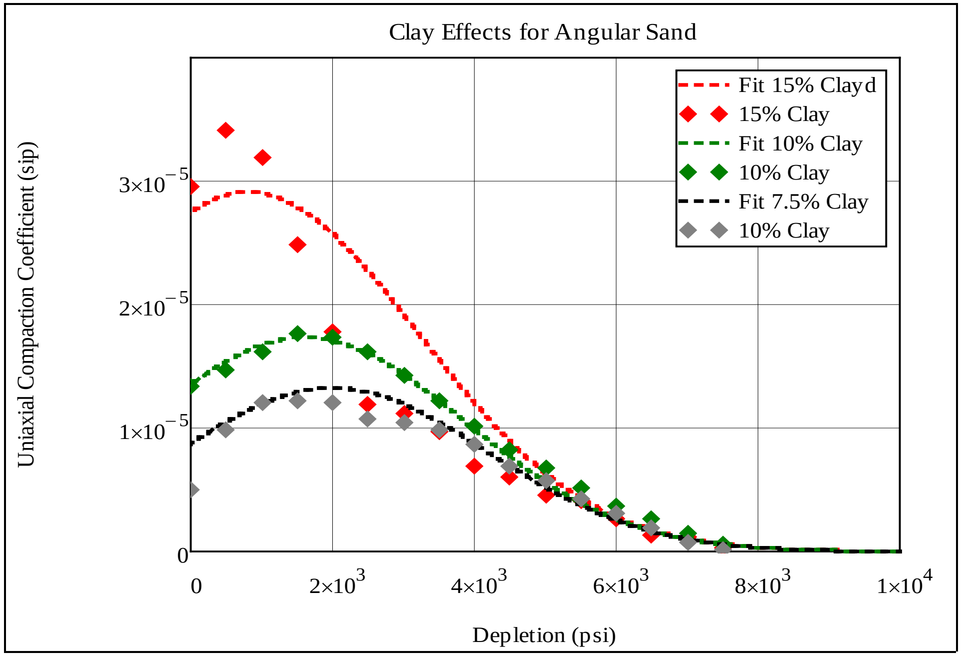
All the individual fits for grain size are performed with the same modeling parameters for the evolution of the network. The only parameter varied between the individual curves is the scaling parameter

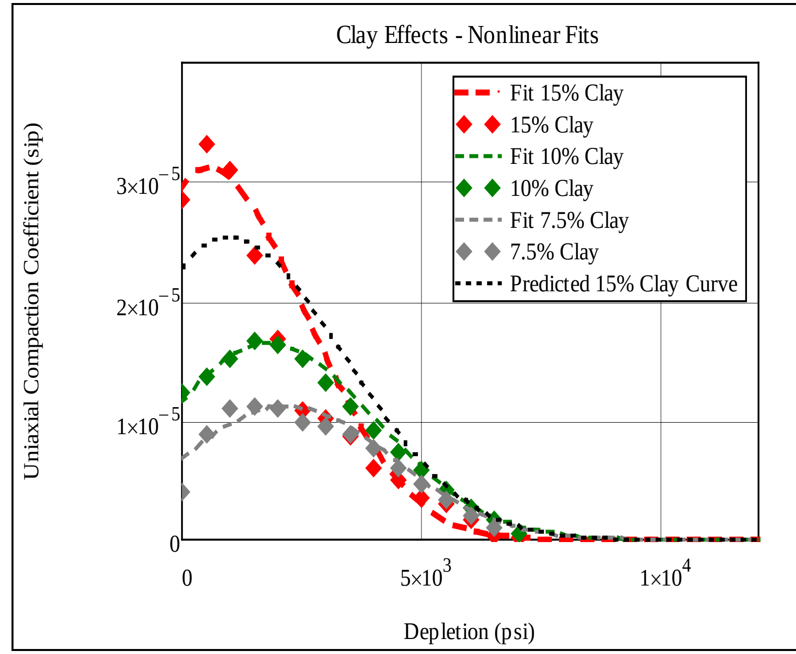
Connection to CAP Data



Combined parameters for the nonlinear fits to the "round" and "angular sands" The CAP estimated grain size is biased by the effects of the 2D nature of a thin section but a linear trend is clearly evident and effects the angular sands and rounded sands in a similar manner.

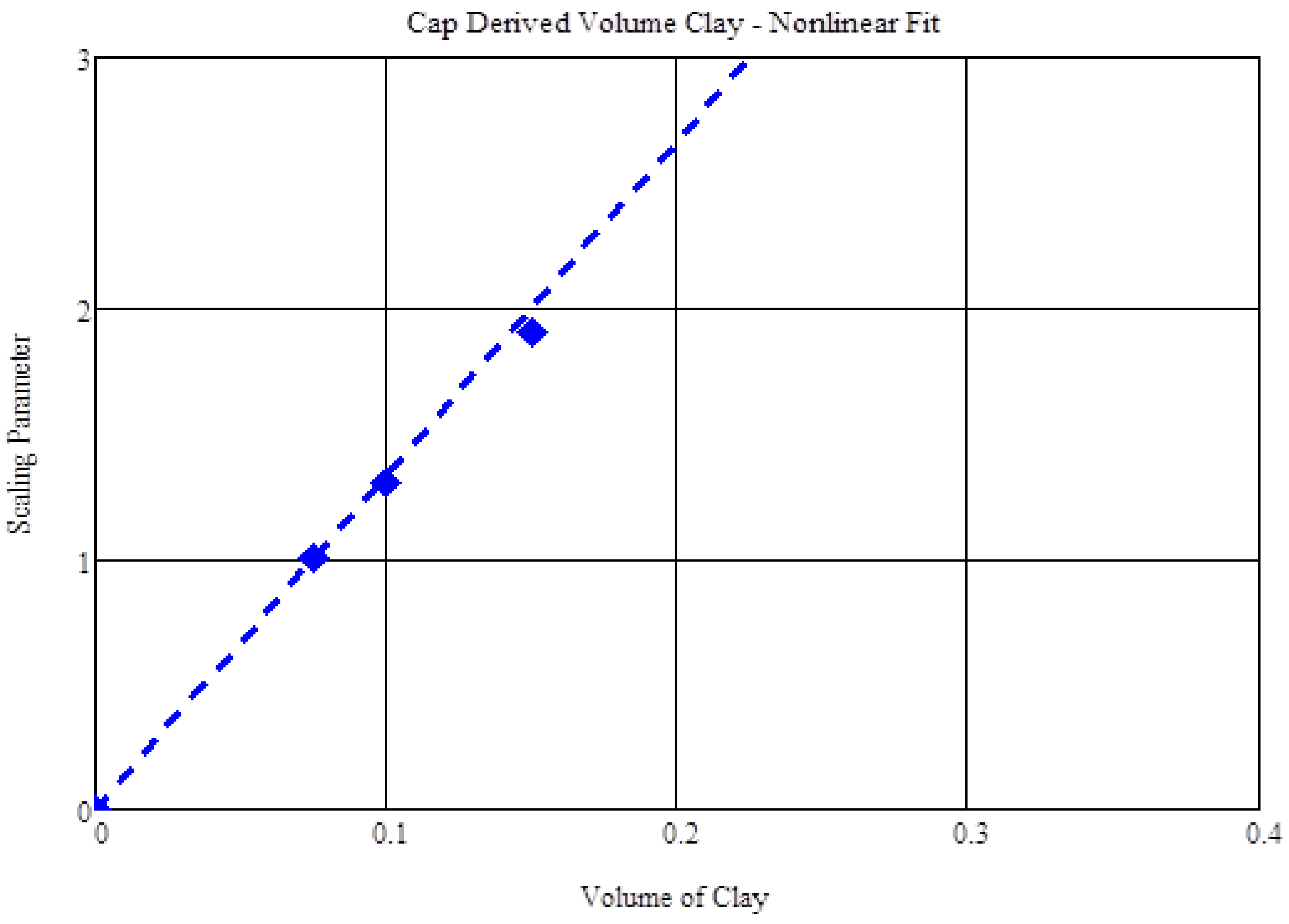
Fits to the Group 1 - Clay Data



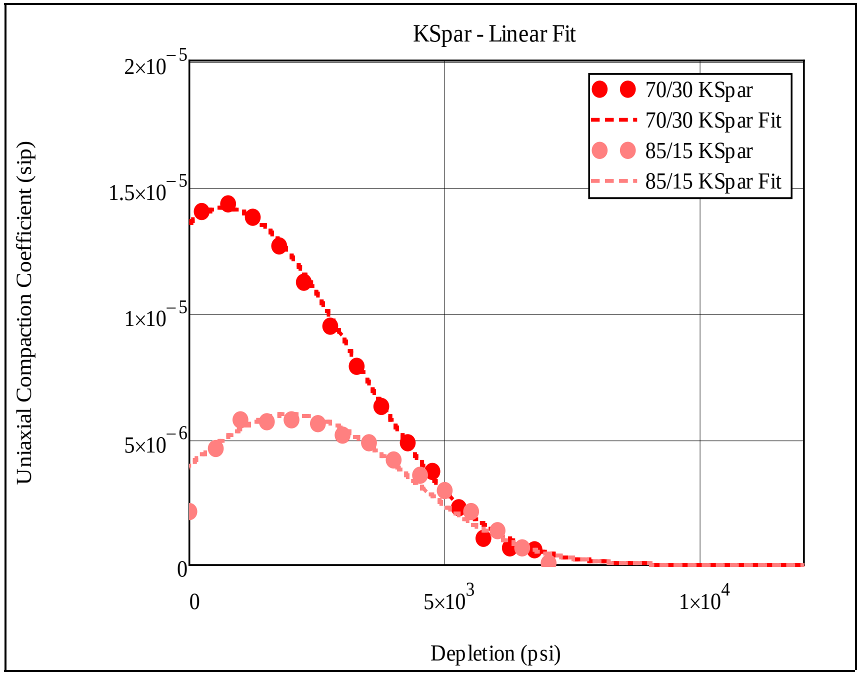


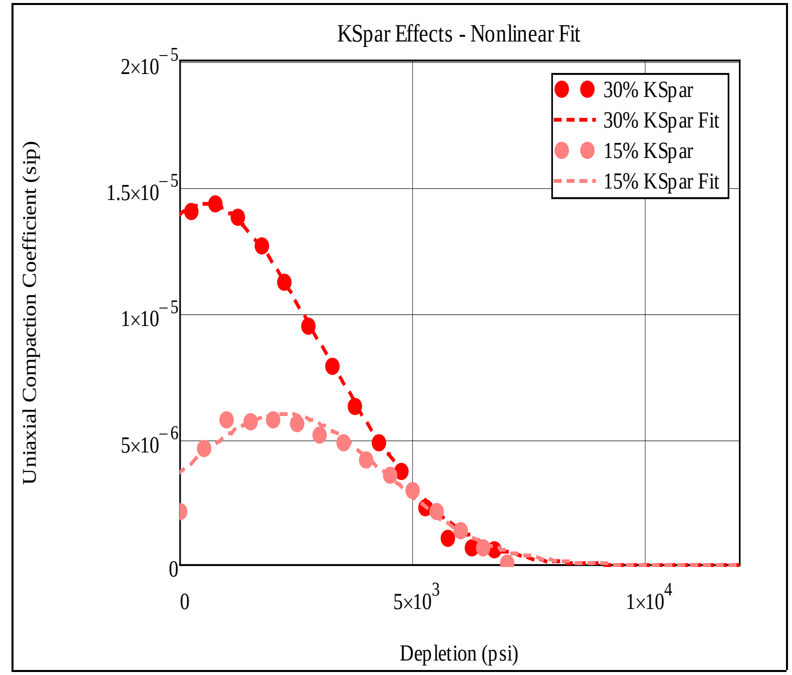
Fits to the clay data. The 7.5% and 10% curves are for the same size inclusions as host grain sizes (all 150 µm). To test the effect of inclusion size the clay clast size was increased to 300 µm, because of the increased compressibility of the clay relative to feldspar significant nonlinear effects are present at 15% clay. The dotted curve is the predicted curve for the intermediate step of including only the volume of clay. The dashed curve shows the results of when the increase in size is added to the calculation. Identical non-linear parameters were used for the clay and the previously shown feldspar data.

Comparison of CAP Derived Volume of Clay to Scaling Parameter



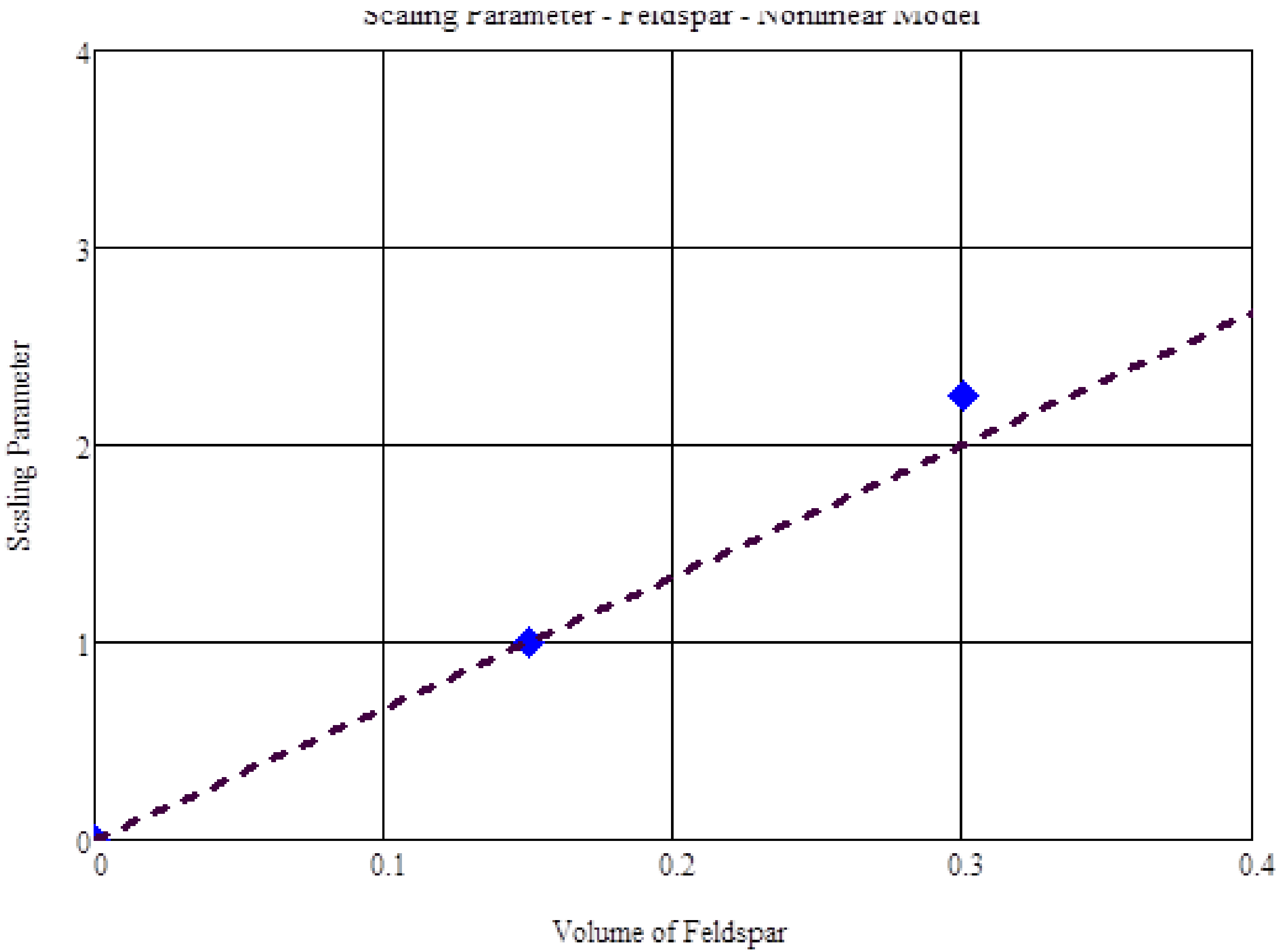
Fits to the Group 1 - Feldspar Data





Data and the fits for inclusion of Feldspar. Both 15% and 30% were measured. The fits are excellent up to 30%. No evidence of the need for a non-linear model is apparent. The same k_o and k_1 are used for each curve, all that varies is the scaling parameter

CAP Derived Volume of Feldspar to Fit Parameter



Group I - Reference Compressibility Curve Parameter Fits

Lithology	Mean Stress (psi)	Standard Deviation (psi)	Peak Value	Baseline C_m (μsips)
Grain Size (150 μm angular)	6900	2400	.036	4.6
Feldspar (15% by weight)	2200	2200	.033	4.6
Clay (7.5% by weight)	2200	2200	.065	4.6

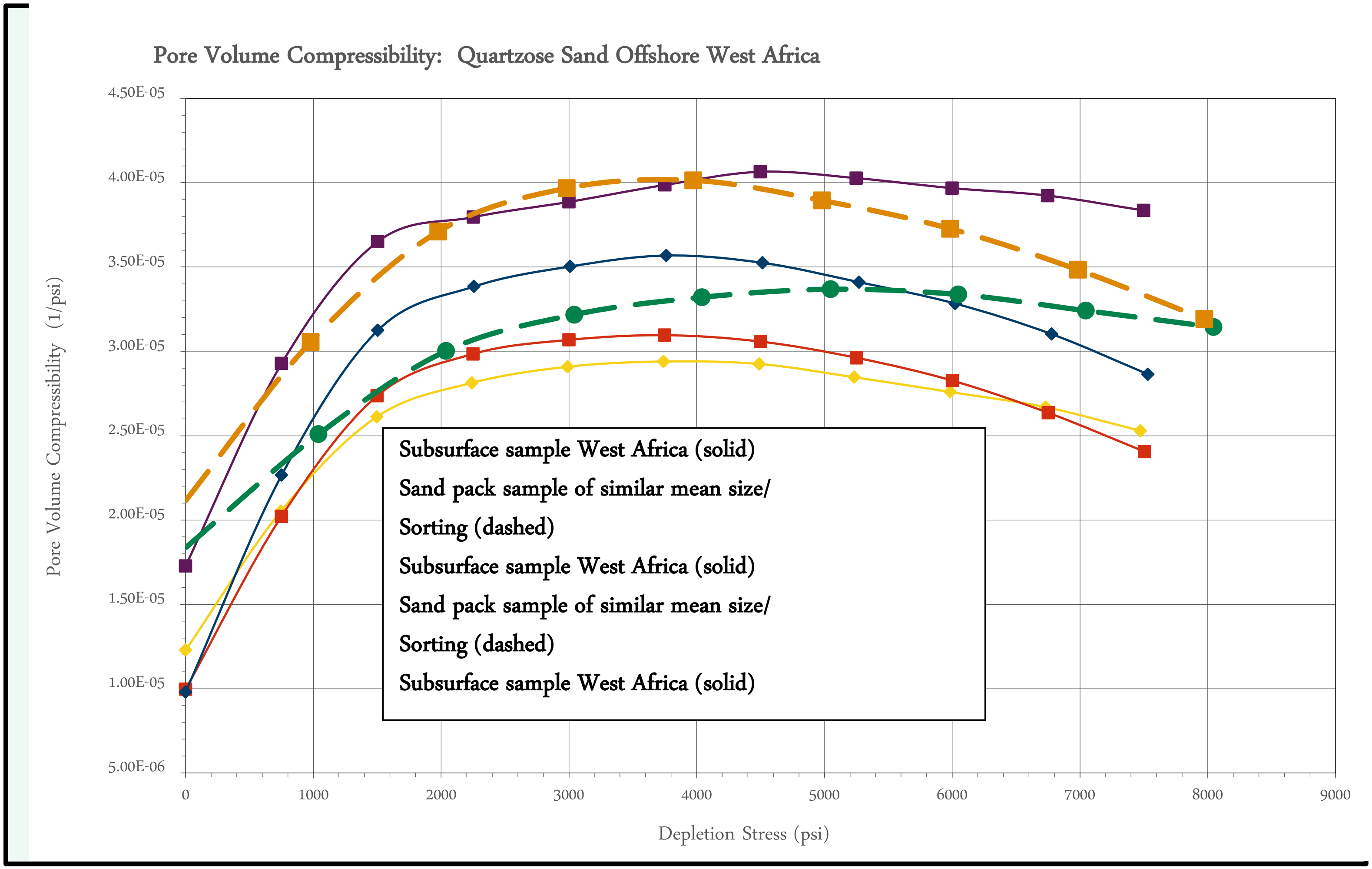
Group I - Compressibility Data

Transport Parameters

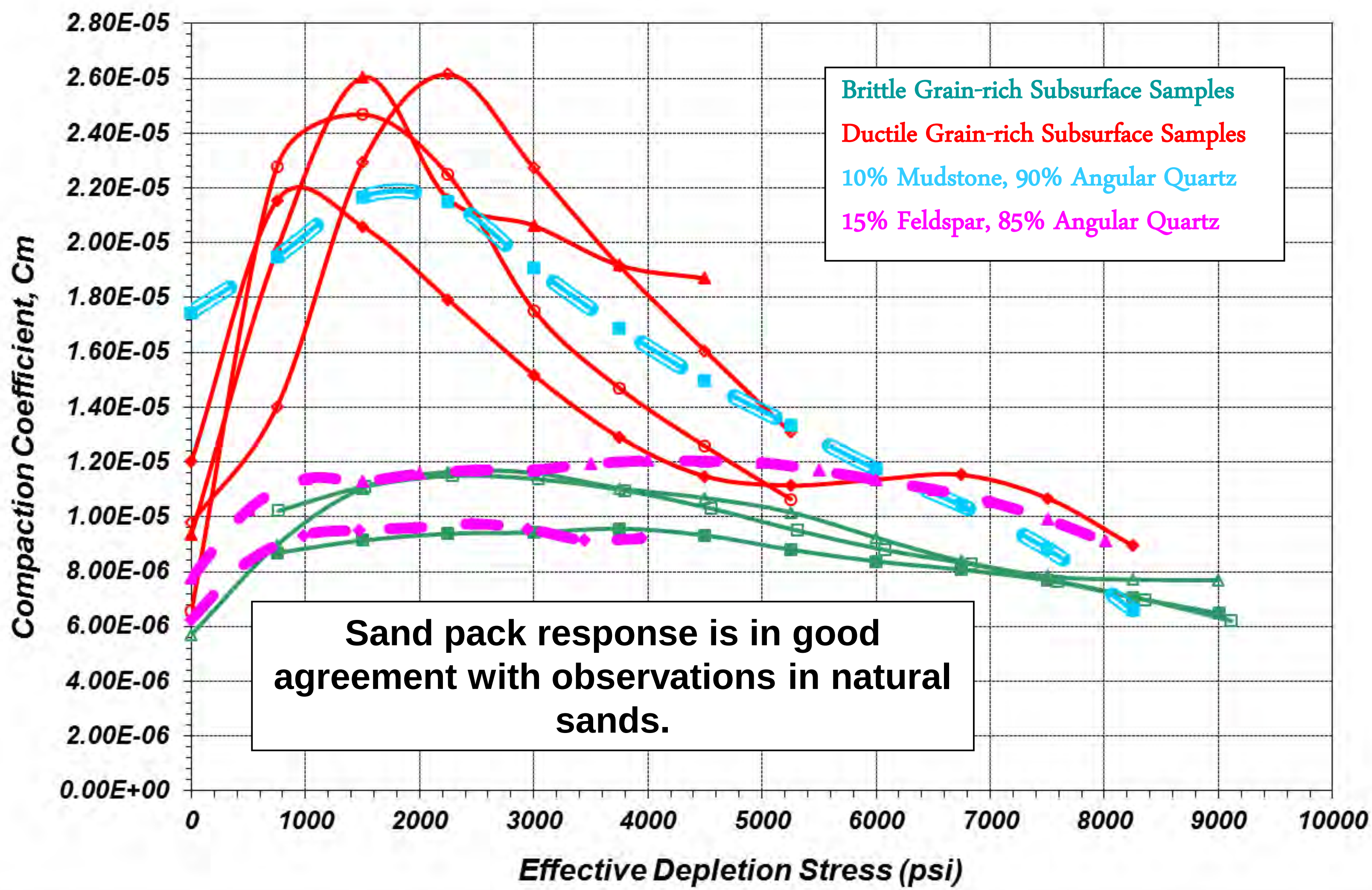
Lithology Change	Curves Fit	Model	k_o	k_1	k_2
Grain Size	5	Linear	-5.0*10^3	.40	0
Feldspar	2	Linear	-1.5*10^3	.10	0
Clay	3	Linear	-1.5*10^3	.10	0
Grain Size	5	Nonlinear	-3.2*10^3	0	-10.5*10^5
Feldspar	2	Nonlinear	-1.0*10^3	0	-10.5*10^5
Clay	3	Nonlinear	-1.0*10^3	.40	-10.5*10^5

The summary tables for the fits to the Group I data. The most significant findings were the much improved fits for the nonlinear model, the ability to fit the data with identical ko and k1 parameters, the strong dependence on the grain size, mineralogy.

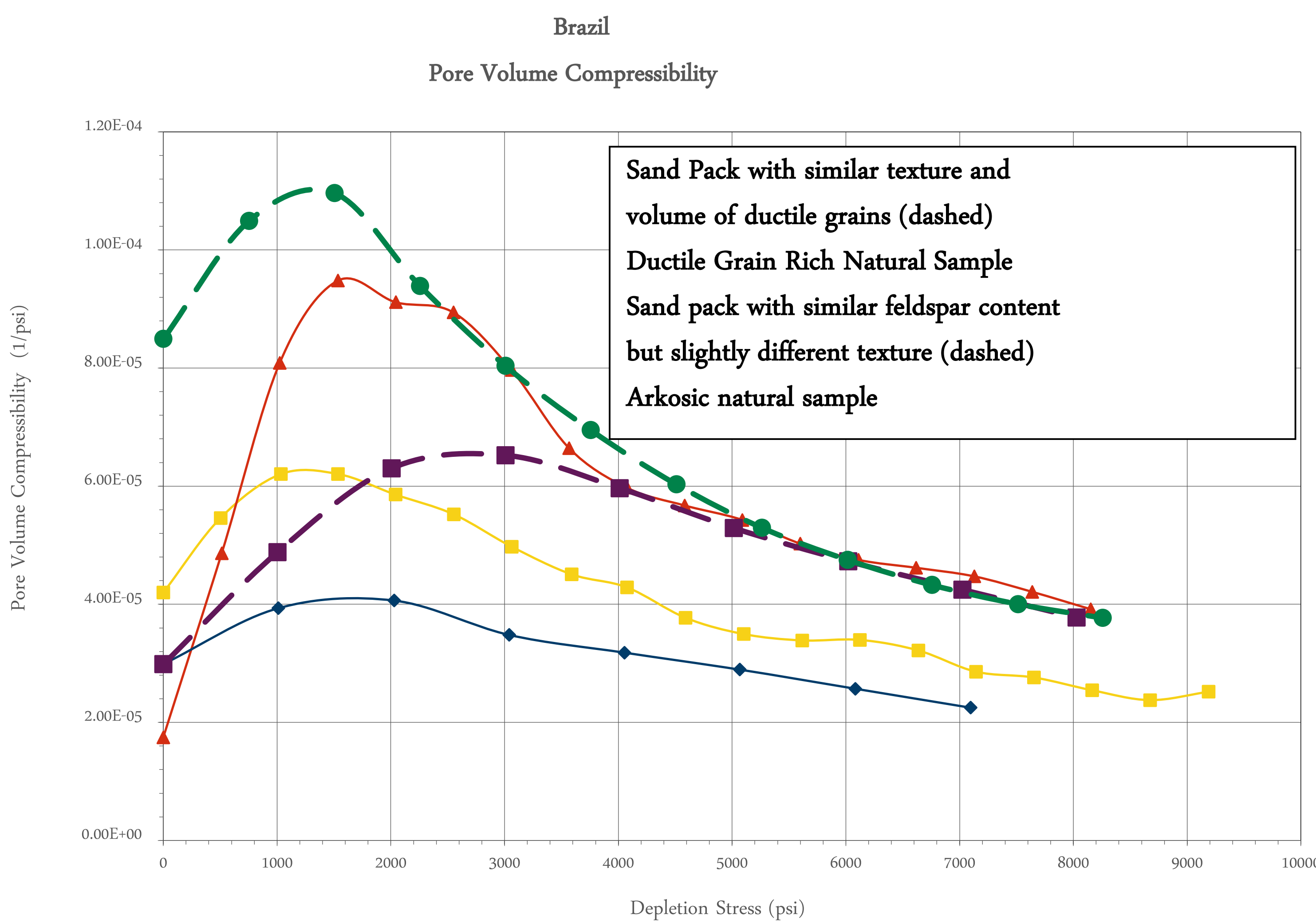
SAND PACK EXPERIMENTS: COMPARISON WITH NATURAL DATA SETS



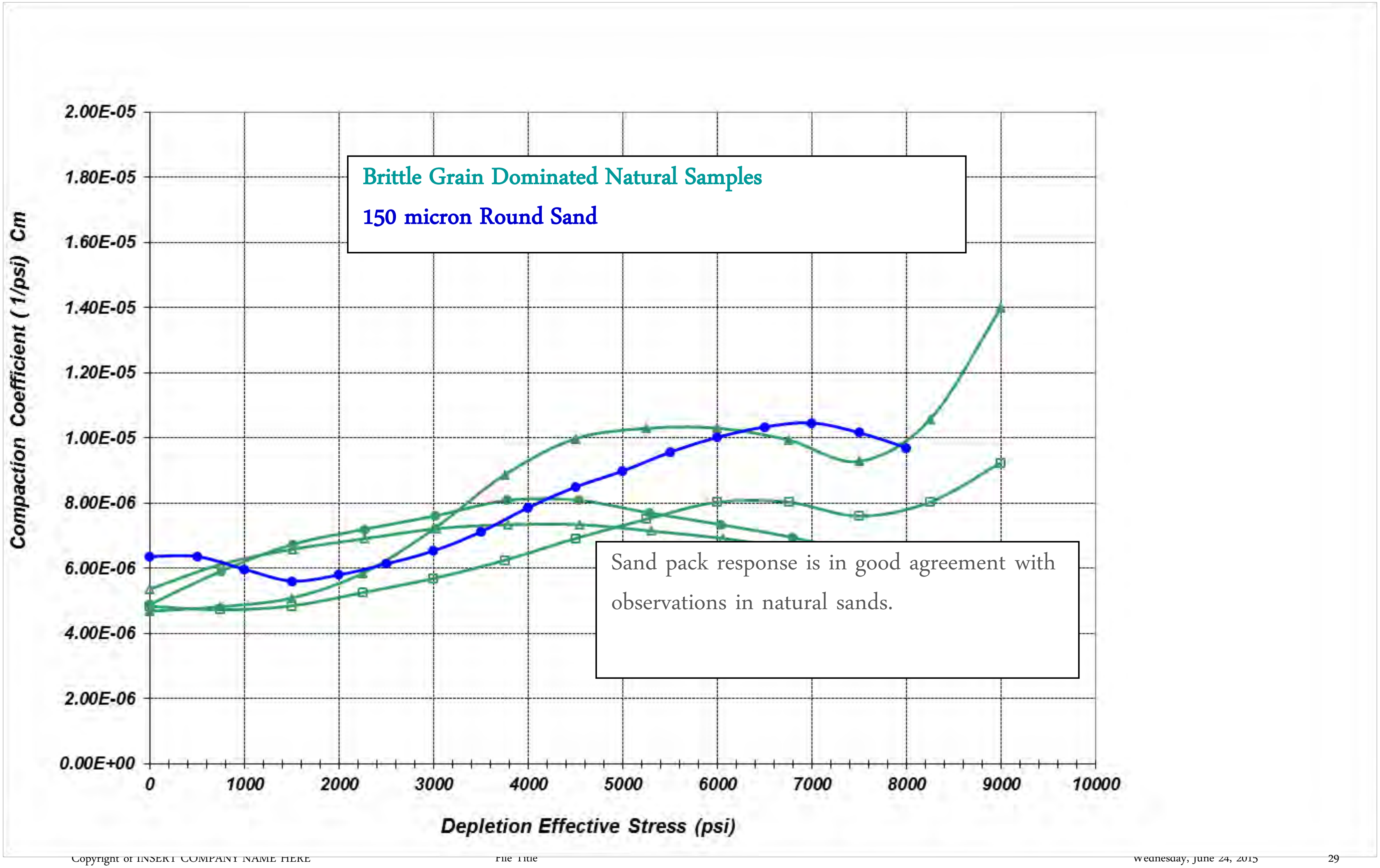
Sand Pack Experiments: Comparison of Group 1 Sand Pack Data with Natural Data Sets



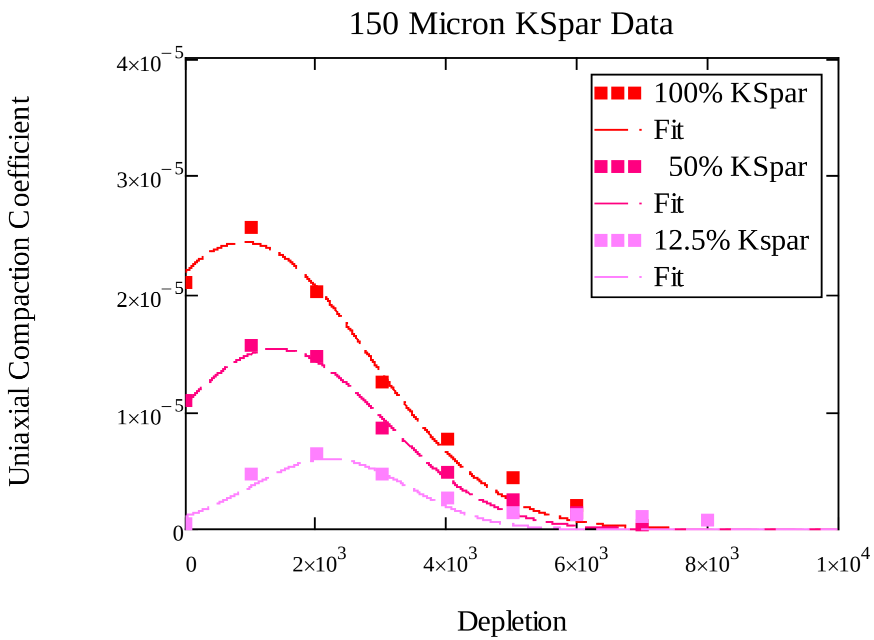
SAND PACK EXPERIMENTS: COMPARISON WITH NATURAL DATA SETS

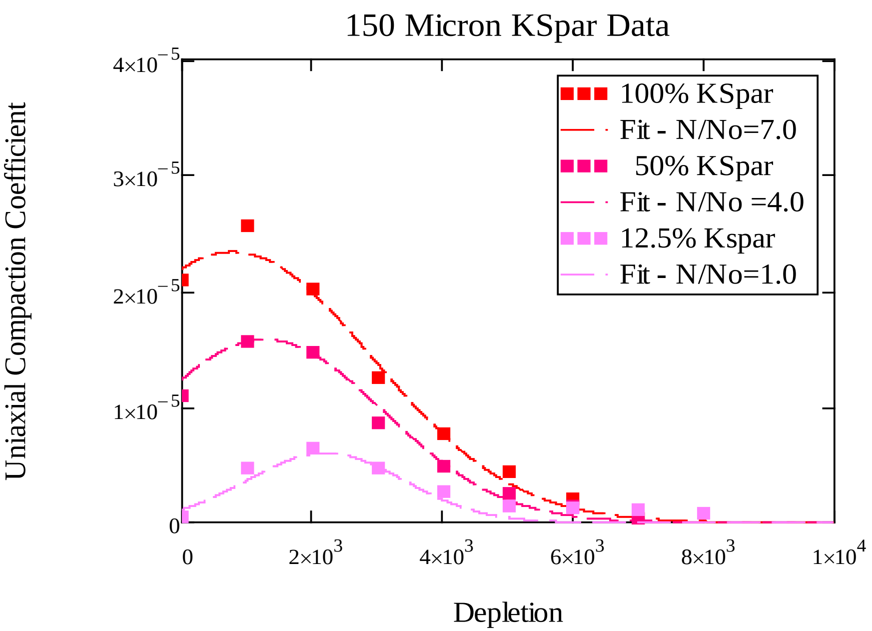


SAND PACK EXPERIMENTS: COMPARISON WITH NATURAL DATA SETS



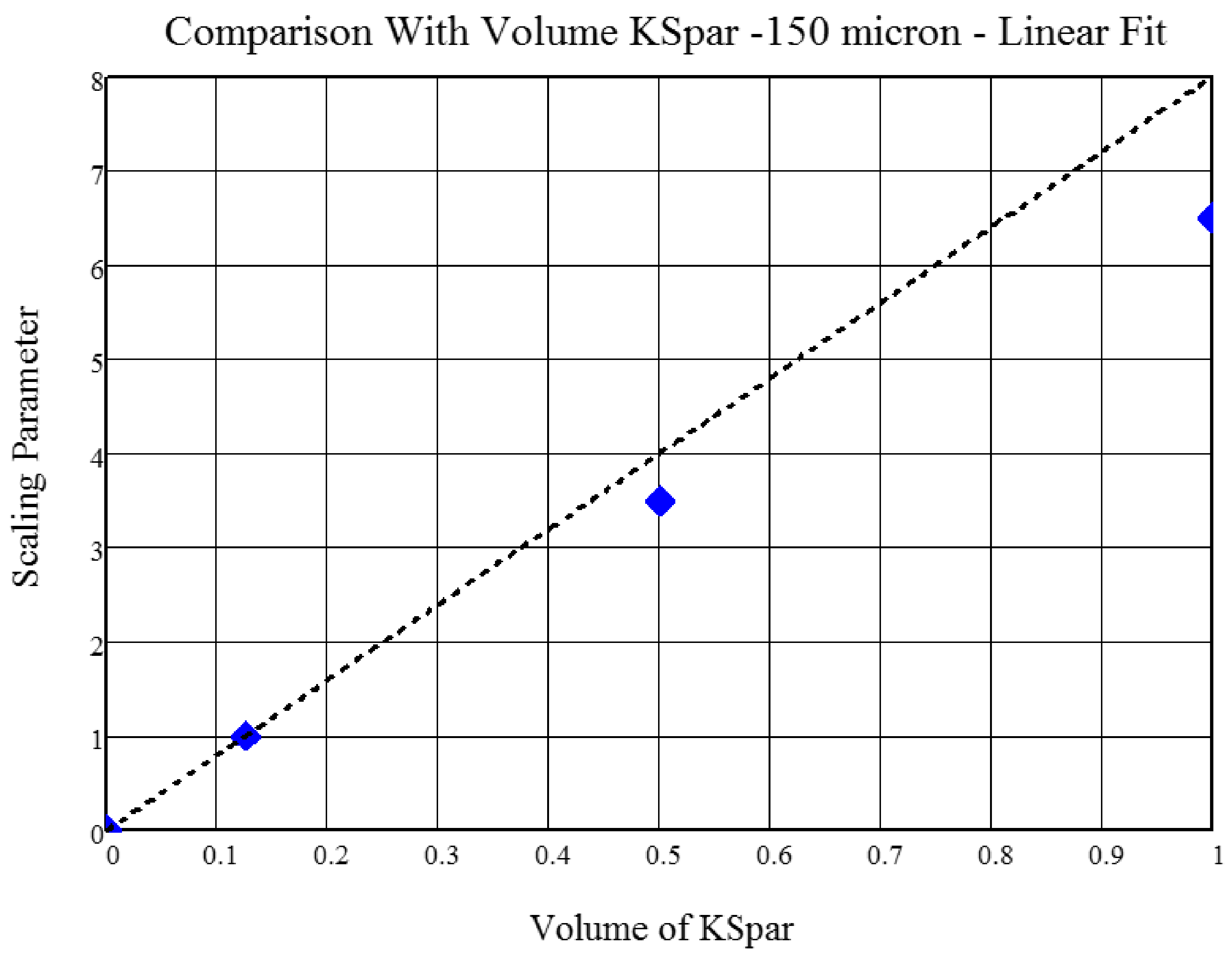
Fits to the Group 2 – 150 µm Feldspar Data





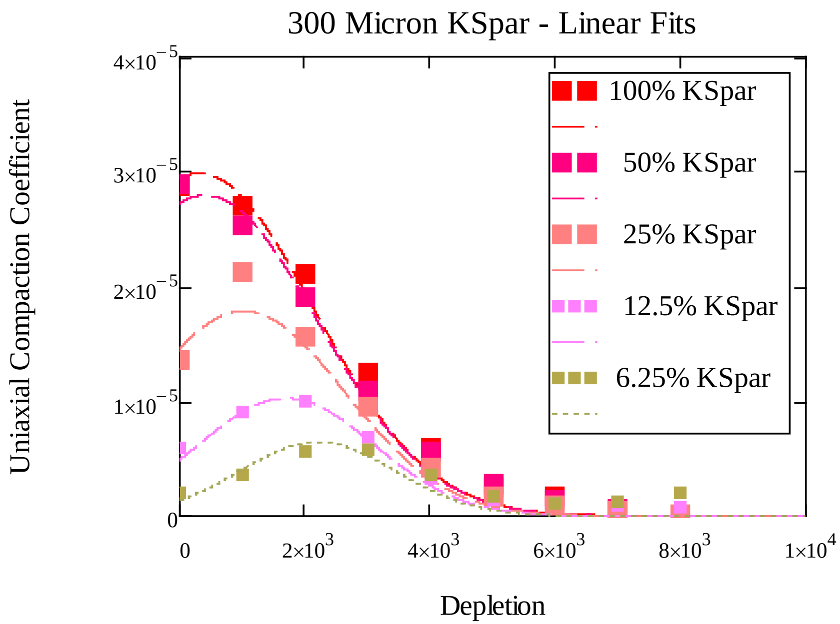
Linear fit to Group II feldspar data. The depletion is in psi. These linear fits are all within experimental accuracy of the data. The implication is that the compacting material never approaches non-linear compaction phenomena.

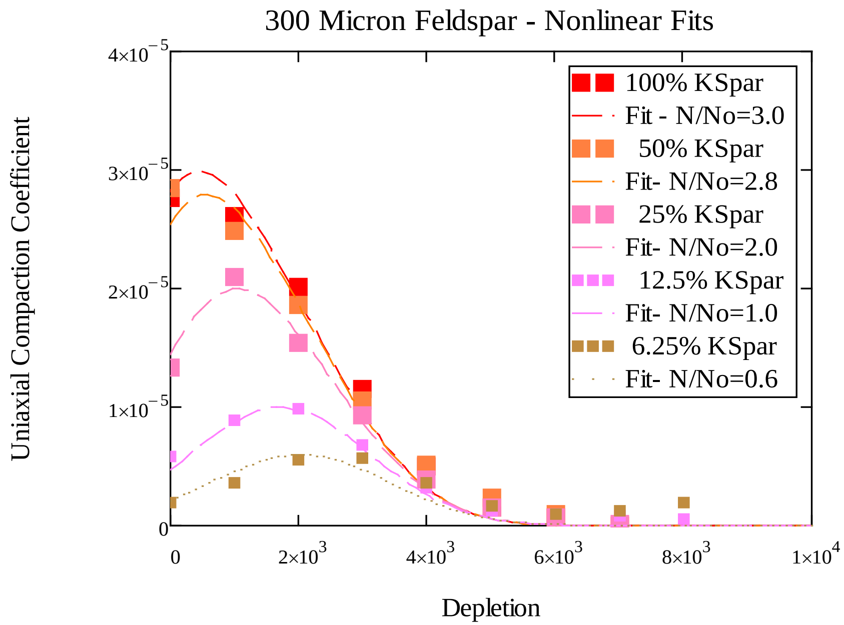
Results of the non-linear fits to the KSpar data. The non-linear fits do not improve the quality at the higher concentrations of Feldspar.



The experimental measurements are all relatively close to the predicted curve. This is explained because of the nearly linear nature of the system. This will not hold true for the larger grained Feldspar or the more brittle Phylite discussed below.

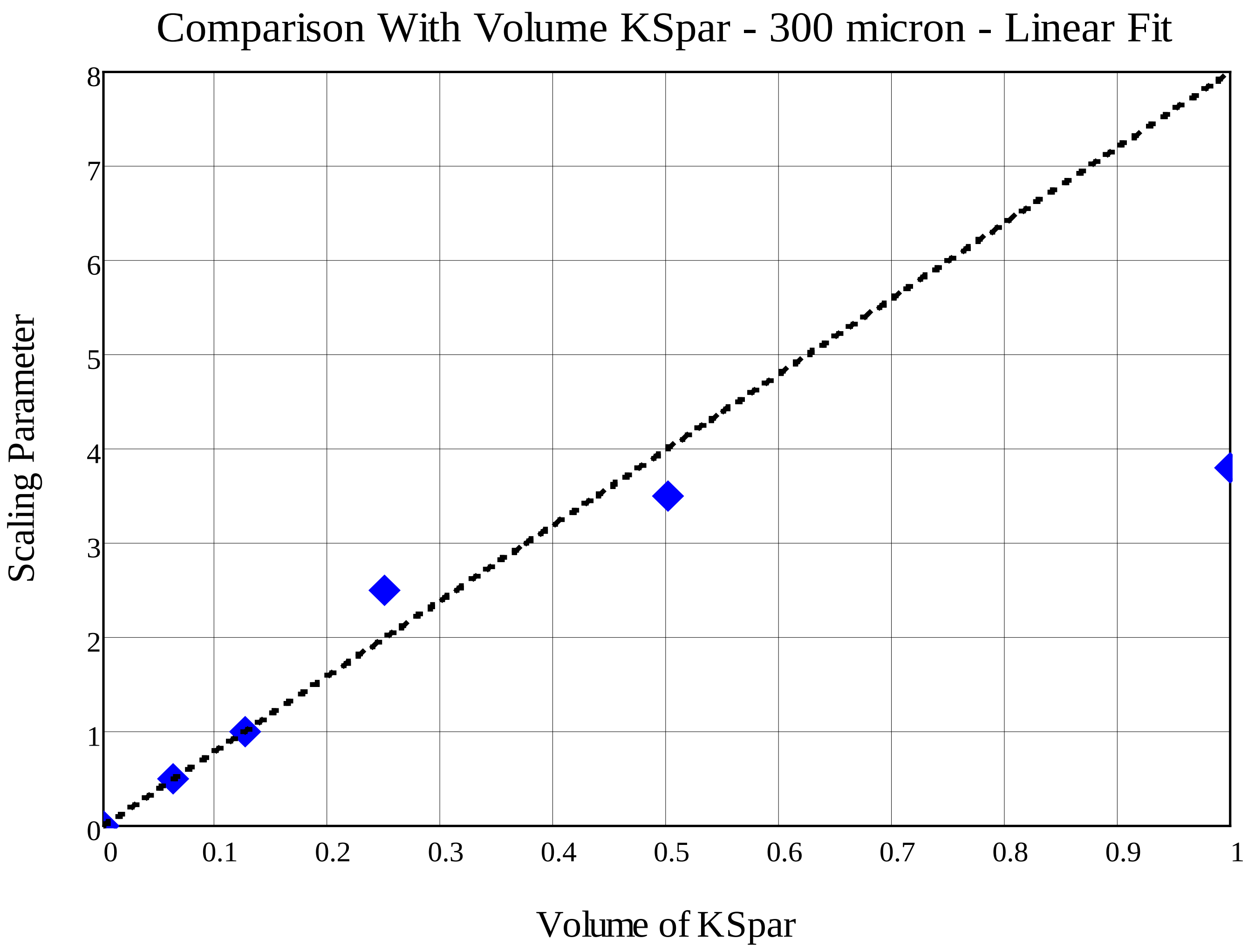
Fits to the Group 2 – 300 µm Feldspar Data





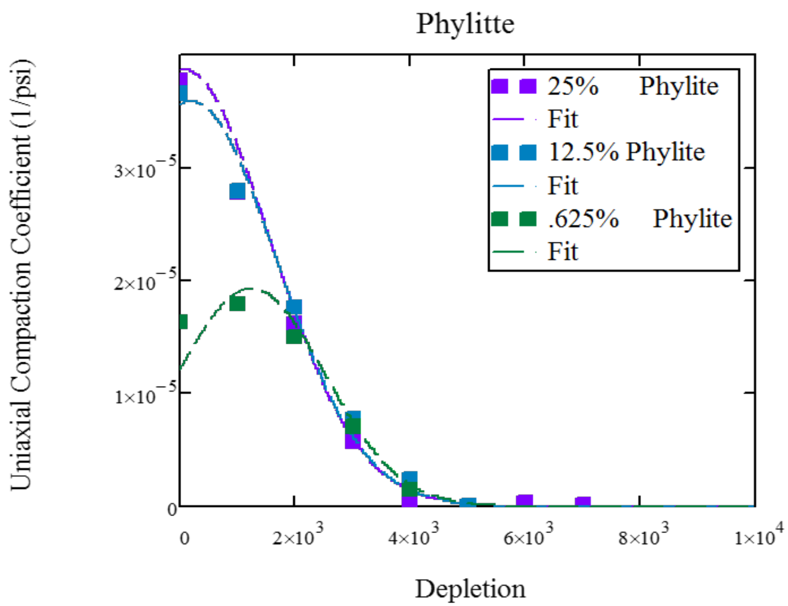
The Feldpar data are shown and fits of the linear model. Here non-linearities become apparent above 12.5% Feldspar where the peak value is no longer honored and the curve becomes too broad. At higher concentrations, the non-linearities become even more apparent

The 300 µm KSpar data and the associated fits. The data shows non-linear behavior above 12.5% Feldspar. Notice the small difference between the curves for the 50% and 100% Feldspar.

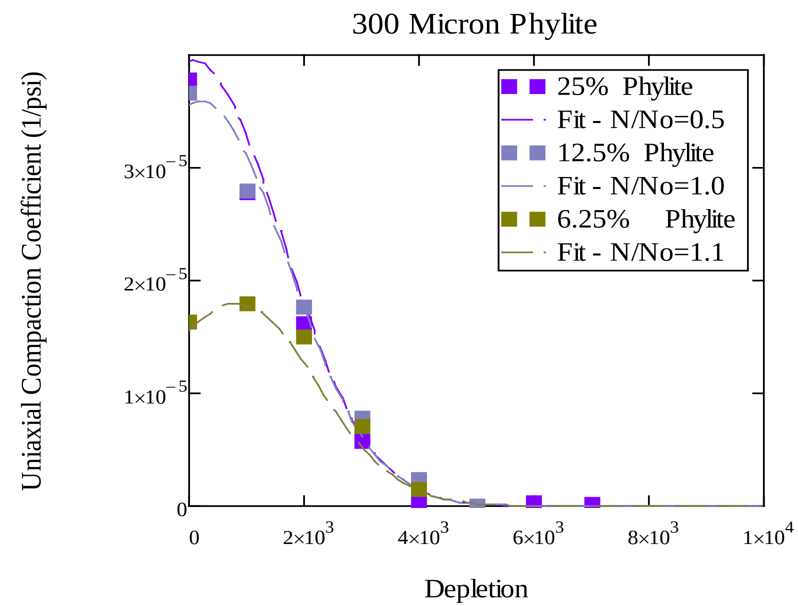


The volume of KSpar and the associated scaling behavior. The rollover in the scaling parameter is thought to be due to the percolation of the feldspar in the sample at high feldspar concentrations it will dominate the compressibility.

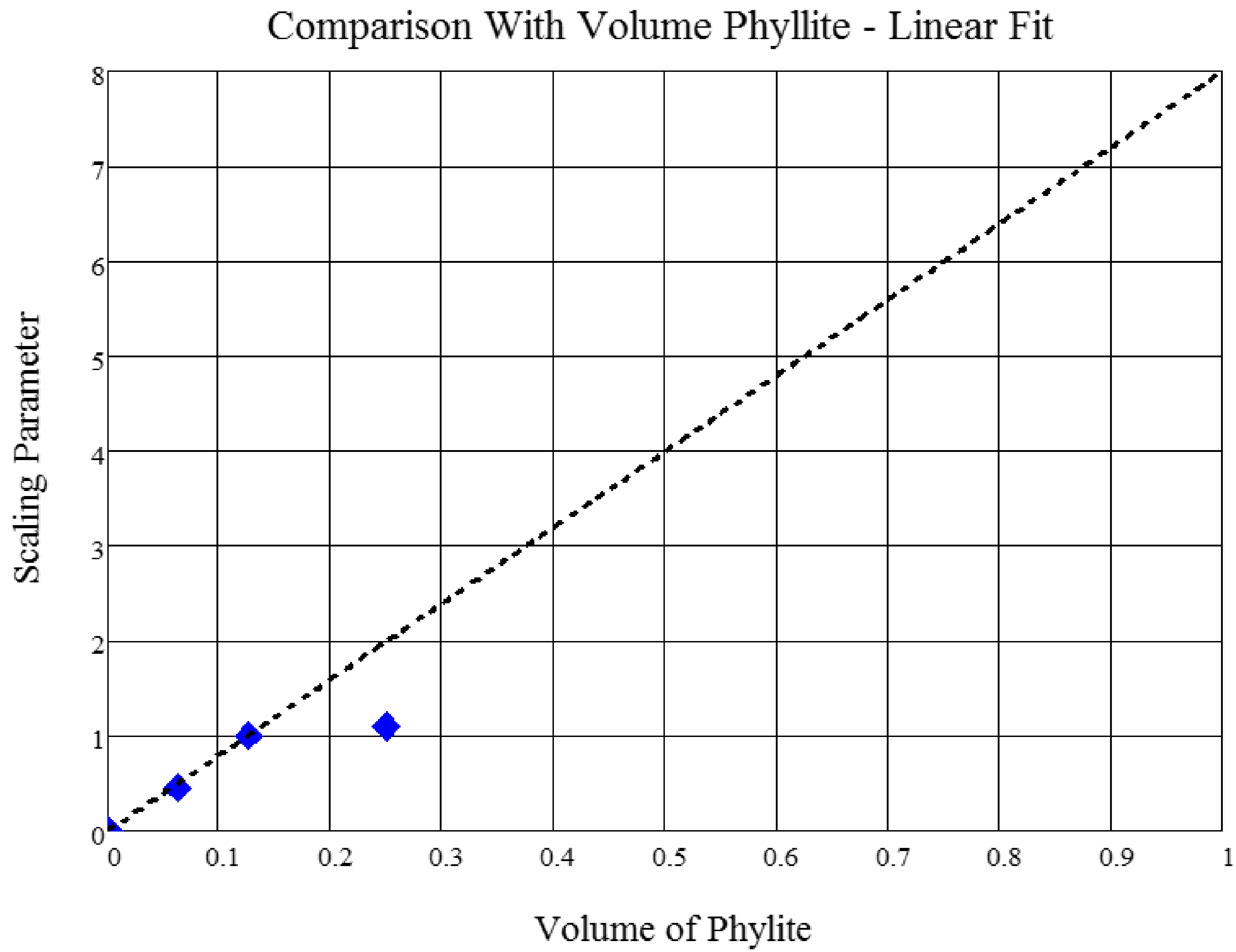
Fits to the Group 2 – 300 µm Phylite Data



Phylite compressibility data and associated fits. The high compressibility and onset of non-linear effects at low concentrations of Phylite is apparent. Small volumes of phylite cause a large increase in compressibility the onset of non-linear behavior occurs at very low volumes of phylite.

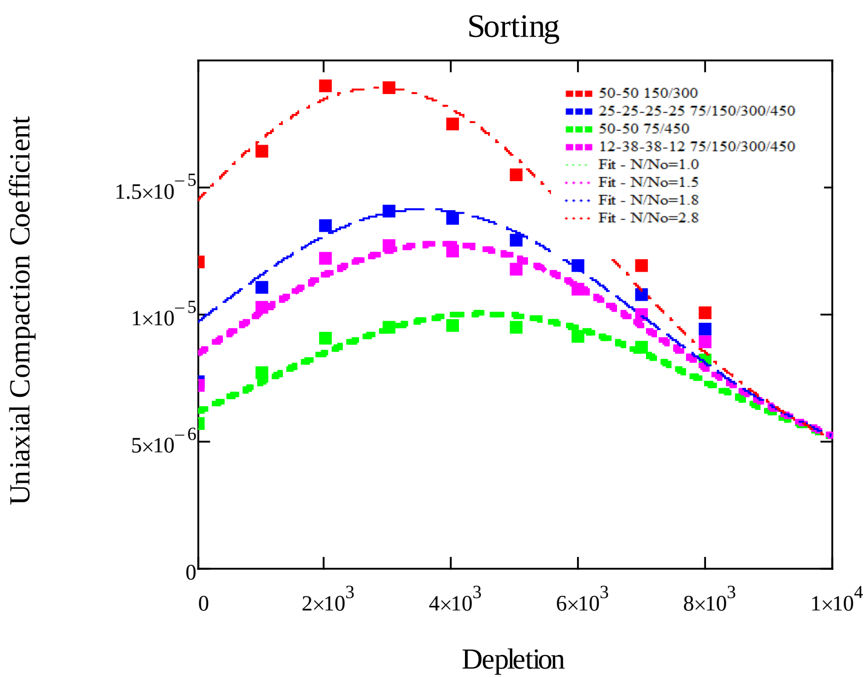


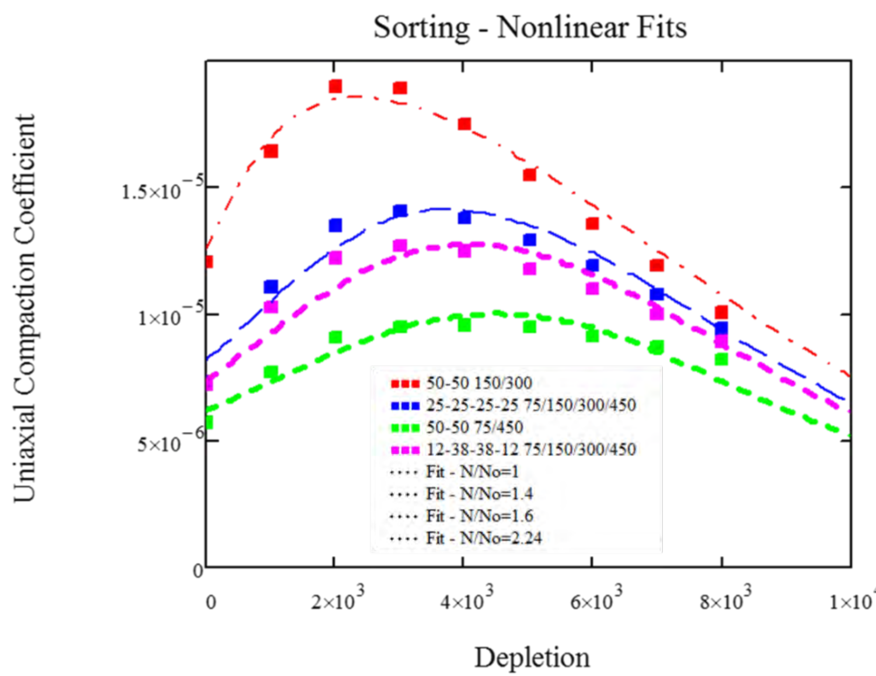
Phylite compressibility data and the associated fits. Phylite has the lowest threshold for compaction of any of the materials tested.



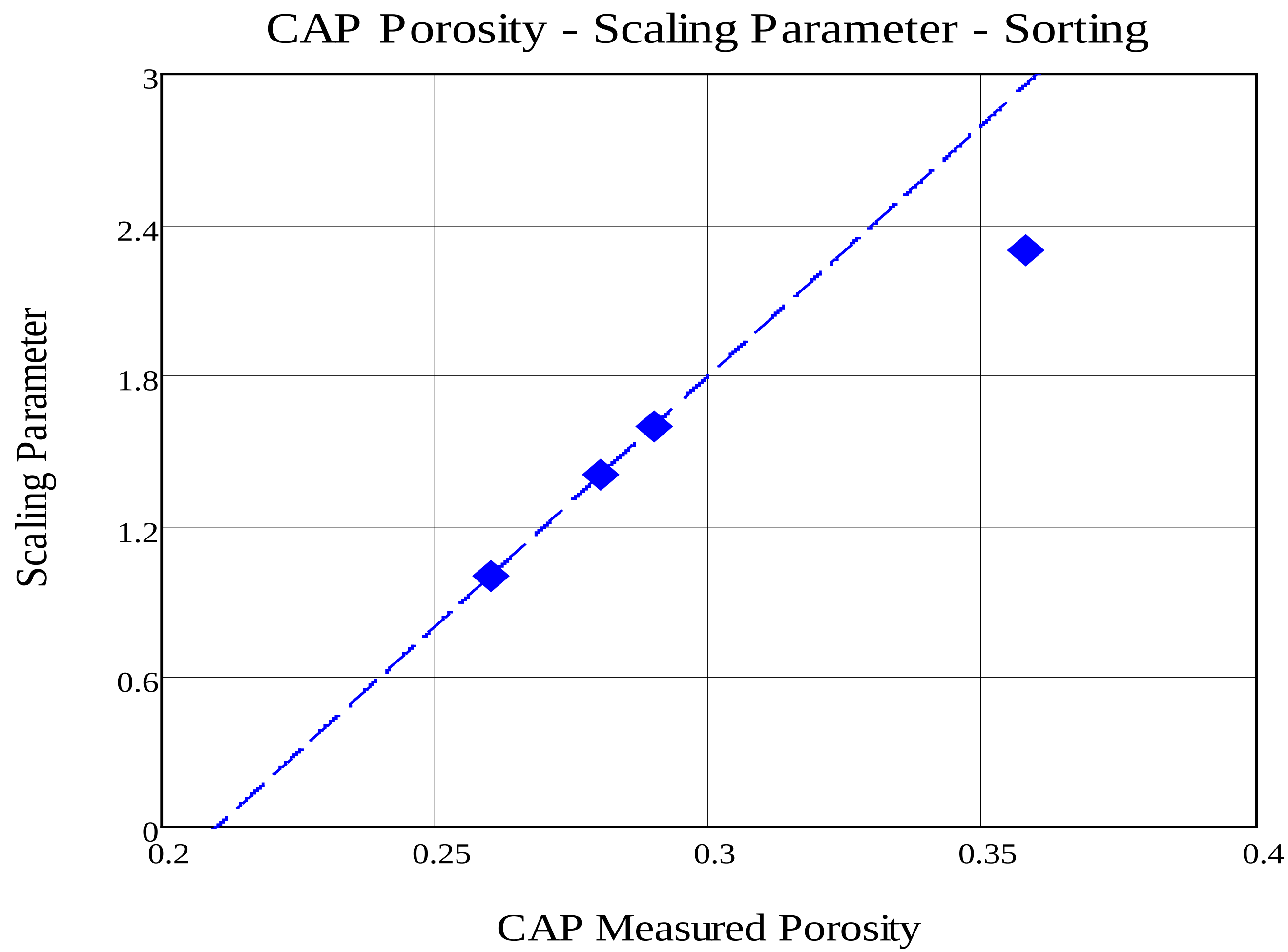
Comparison of the model scaling parameter with the CAP derived volume of Phylite. The addition of Phylite causes a large increase in compressibility. Similar to the 300 mm KSpar data the roll over in the scaling parameter is thought to be due to the onset of phyllite grain breakage below the initial test stress. This starts to occur at about 10% volume of phylite.

Fits to the Group 2 – Sorting





Sorting data and linear model fits. For the higher compressibility data the linear compressibility models fail to capture the skewing of the curve evident at low stress. The sorting varies from most compressible: 50%/50% - 150 □m/300 □m mix. Then the 25%/25%/25%/25% - 75 mm/150 mm/300 mm/450 mm mix, the 12%/38%/38%/12% - 75 mm/150 mm/300 mm/450 mm mix, to the least compressible 50%/50% - 75 mm/450mm mix. To minimize size effects the peak of the grain size distribution was kept nearly constant. It is expected that a more detailed look at parameters such as contact length and grain coordination number will help to understand the variation in compressibility.



Correlation between the CAP measured porosity and non-linear model scaling parameter. The values for the scaling parameter are virtually identical to the linear model parameters. The reference curve is the 50%/50% - 75 □m/450 □m sorting data. The change in peak compressibility for this data is about 10 msips much less than the change in compressibility change for 15% feldspar or clay.

Group II Reference Curve Data

These curves were all for 12.5% inclusions

Lithology Change	Mean Stress (psi)	Standard Deviation (psi)	Peak Value (msips)	Baseline C_m (msips)
Feldspar 300 µm	1700	1400	.036	10
Feldspar 150 µm	2200	1200	.018	10
Phylite	200	1500	.135	10

Group II Compressibility Data

Lithology Change	Curves Fit	Model	k_o	k_1	k_2
300 μm Feldspar	5	Linear	-1.25*10^3	.20	0
150 μm Feldspar	5	Linear	-1.20*10^3	.10	0
Phylite	3	Linear	-1.25*10^3	.30	0
300 μm Feldspar	5	Nonlinear	-.55*10^3	0	-10.*10^6
150 μm Feldspar	3	Nonlinear	-.55*10^3	0	-10.*10^6
Phylite	3	Nonlinear	-.55*10^3	0	-10.*10^6

Conclusions

- Sand pack studies allow separation of individual textural and mineralogic parameters for study of their influence on production time scale compaction.

- Coarser grained sands are more compressible than their finer grained equivalents.

- As grain angularity increases, compressibility increases.

Conclusions

- Small volumes of load bearing ductile grains increase overall compressibility and lower the depletion stress at which maximum compressibility is achieved Strain mechanisms include plastic flow of ductile components and enhanced brittle failure of competent grains

- As volume and size of ductile grains increase compressibility continues to increase with attendant decrease in depletion stress required to achieve maximum Cm

- Plastic response to stress becomes dominant over brittle grain failure as volume of ductile grains increases above 15%.

Future Work

- Apply model to subsurface data sets from several basins
  - How global is the model?
  - Is a separate fit required for each basin?
- How can stress and cementation history be incorporated into our modeling?
- Can the model be extended to geologic time scales?
- Develop stress dependent permeability and velocity models
- Implement nonlinear model